Note that you have to answer any 10 out of the 20 questions below.

Entrance Examination - Part B
Department of Computer Science and Engineering, IIT Madras

Max. Marks: 60 Duration: 120 Minutes

Please read the Instructions given in the first page of the Answer Booklet carefully.

Questions

1. A *Permutation* of the given sequence $(a_1, a_2, \cdots, a_n)$ is *stack generatable* if we `PUSH` the numbers $a_1, a_2, \cdots, a_n$ in that order using a stack intermixed with `POP` operations and obtain the permutation. For example, the permutation $(2, 3, 5, 4, 1)$ of the given sequence $(1, 2, 3, 4, 5)$ is stack generatable because it can be generated by the following sequence of operations: `PUSH(1), PUSH(2), POP, PUSH(3), POP, PUSH(4), PUSH(5), POP,POP,POP`. The outputs obtained from the five `POP` operations are $2, 3, 4, 5, 1$ in that order. Construct a sequence of `PUSH/POP` operations that would generate the following permutation:
$$(4, 6, 5, 7, 3, 9, 8, 2, 10, 1, 11, 13, 12)$$

2. Consider the AVL-Tree (Balanced Binary Tree) given below:

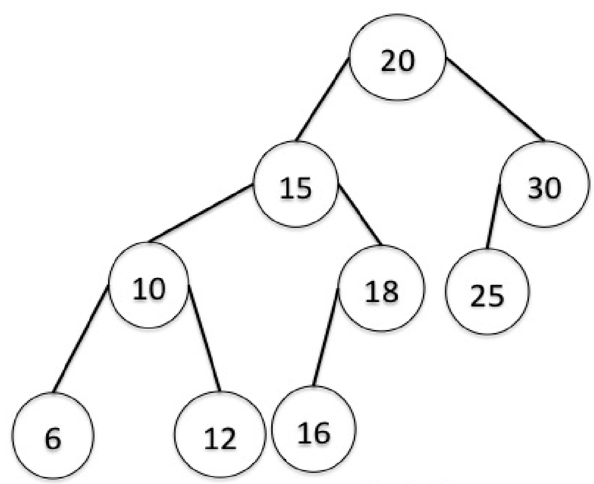


Fig (a)

(a) the Balance number of nodes containing $[6, 10, 12, 15, 16, 18, 20, 25, 30]$ are $[-, -, -, -, -, -, -, -, -]$
(b) Draw the AVL-Tree obtained after deletion of the node 30.
(c) the Balance number of nodes containing $[6, 10, 12, 15, 16, 18, 20, 25]$ after deleting 30 are $[-, -, -, -, -, -, -, -, -]$

3. Consider the following program:

```
int x = 1;
void g() { printf("%d",x); x = 2; }
void f() { int x = 3; g(); }
main() { f(); printf("%d",x); }
```

   (a) What is the output of the above program with static scoping rules?

   (b) Suppose that `C` supports dynamic scoping, what is the output of the above program with dynamic scoping rules?

4. Express the following in predicate calculus.

   (a) There exists a cricketer who everyone admires.

   (b) Every student has a friend who got an A grade in the AI course.

5. An 8×1 multiplexer has inputs A, B, and C connected to the selection inputs s_2, s_1, and s_0, respectively, where A and C are the *most-significant* and *least-significant* selection lines respectively. The data inputs, I_0 through I_7, are as follows: $I_1 = I_2 = I_7 = 0$; $I_3 = I_5 = 1$; $I_0 = I_4 = D$; and $I_6 = D'$. Determine the Boolean function that the multiplexer implements.

6. Consider the following function of 4 Boolean variables:

$$Z(A,B,C,D) = \overline{A}.B.\overline{C} + A.\overline{C}.D + B.C.D + \overline{B}.\overline{D}$$

   (a) Give the minterm expression for Z.

   (b) Derive the minimal sum of products (MSOP) expression for Z using Karnaugh map.

   (c) Give a circuit that uses only NAND gates to realize the MSOP expression for Z in Part (b). Assume that the 4 variables A, B, C, and D, and their complements $\overline{A}$, $\overline{B}$, $\overline{C}$ and $\overline{D}$ are available as input to the circuit.

7. For a pipelined processor, match the following. Note that the matching is possibly of many-to-one type.

A. Cache miss	1. Stack
B. Read-after-write data dependency	2. Multi-cycle operation
C. TLB miss	3. Static branch prediction
D. Floating point arithmetic instruction	4. Suspension of process
E. Control hazard	5. Hardwired control unit
F. Page fault	6. Operand forwarding
G. Interrupt from an I/O device	
H. Interrupt from CPU scheduler	
I. RISC architecture	
J. Function call	

8. Provide an example to illustrate Belady's anomaly in a FIFO page replacement policy, i.e., increasing the number of page frames also increases the number of page-faults.

9. (a) Is $\lfloor \log n \rfloor!$ bounded by a polynomial?

   (b) Is $\lfloor \log n \log n \rfloor!$ bounded by a polynomial?

   State and prove your claim

10. Let A and B be regular languages. Show that their concatenation $A \cdot B = \{xy | x \in A, y \in B\}$ is also a regular language.

11. For a string $x \in \{0,1\}^*$, let x^{R} denote the string reverse of x, and $\bar{x}$ be the string obtained from x by changing all 0's to 1's and 1's to 0's. Show that the set

$$A = \{x \mid x^{\mathsf{R}} = \bar{x}\}$$

   is **context free**.

12. One way of making linear regression applicable more widely is to use basis expansions, i.e., adding more features to the input set. Suppose that the data is described by a p-tuple, $(x_1, x_2, \ldots, x_p)$. Comment on the utility of the following sets of features. Specifically describe the family of functions that can be represented by a linear combination of these features.

   (a) $\left(x_1, \ldots, x_p, x_1^2, x_1x_2, x_1x_3, \ldots, x_1x_p, x_2^2, x_2x_3, \ldots, x_p^2\right)$

   (b) $\left(x_1^2, x_2^2, \ldots, x_p^2\right)$

   (c) $(x_1 + x_2, x_1 + x_3, \ldots, x_1 + x_p, x_2 + x_3, x_2 + x_4, \ldots, x_2 + x_p, \ldots, x_{p-1} + x_p)$

   (d) $(x_1, x_1 + x_2, x_1 + x_2 + x_3, \ldots, \Sigma_{i=1}^{p} x_i)$

13. An item can be pushed (using the operation `push`) onto a stack, only if it has been initialized (using the operation `init`) before. An item can be popped (using the operation `pop`) from a stack only if the stack is not empty. The top element of the stack can be checked (using the operation `top`) only if the stack is not empty. Write the grammar to parse a series of successful stack operations. For example, a sequence of successful stack operations can be `init, push, push, pop, push, pop, top, pop`.

14. Write a SDT to count the number of 1s in the binary string derived from the following grammar.

```
B -> B 1 | B 0 | 1
```

15. Consider the following relations with key (`studentId, subjectId`).

    `Enrollment(studentId, subjectId, finalMarks)`

    On this relation, express the following query using relational algebra.

    Get the Id of students who *never got more than eighty marks* in any subject they enrolled.

16. Two users, one using `telnet` and another sending files with `ftp`, both send their traffic out via a router R. The outbound link from R is slow enough that both users keep packets in R's queue at all times. Discuss the relative performance seen by the `telnet` user if R's queuing policy for these two flows is

    (a) round-robin service;

    (b) fair queuing; and,

    (c) modified fair queuing, where we count the cost only of data bytes, and not IP or TCP headers.

    Consider outbound traffic only. Assume `telnet` packets have 1 byte of data, `ftp` packets have 512 bytes of data, and all packets have 40 bytes of headers.

17. Plot the curve:

$$y = (x - 1)^2 + 4$$

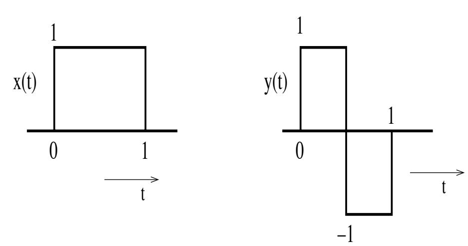
18. In a certain city three car brands, A, B, C have 20%, 30% and 50% of the market share, respectively. The probability that the care needs major repair during the first year of purchase for the three brands is 5%, 10%, 15%, respectively.

    (a) What is the probability that a car in this city needs major repair during its first year of purchase?

    (b) What is the probability that a car requiring major repair during its first year of purchase is from manufacturer A?

19. An experiment is performed n times, in each of which an event X may occur with a probability p. Determine $E[\bar{X}]$ and $Var(\bar{X})$, where $\bar{X}$ is the complement of the event X.

20. Given the signals $x(t)$ and $y(t)$ as shown in the figure below:



    (a) Sketch $x(t) + y(t)$ and $x(t) - y(t)$.

    (b) Compute the energies of these two signals.